Simply from Sharp to Smooth

Background
Everything is a surface: When we see something, we see its surface. When we design something on a computer, we describe it by its surface. When we manufacture something, we generate its surface by milling or obtain it by other techniques. Today, most things that are manufactured are designed in a computer. Thus we need to speak geometry to a computer and that means we need algorithms by which we can transform a concise description of a surface into the surface itself.

An appealing simple technique for that task is called subdivision where we refine polyhedral meshes to obtain arbitrarily fine meshes representing what we want. A particular intuitive subdivision technique is called edge or corner cutting. It means just that: Given a polyhedron, we chop off its edges by which we make the polyhedron smoother and smoother while we introduce more and more new and less sharp edges. While this idea is simple, it is much harder to analyze and control what comes out of it. Yet, one thing is straightforward: Cutting edges preserves convexity and if we do not cut any faces completely, we generate convex surfaces with given tangent planes. In the dual world this means that edge cutting is a way to generate convexity preserving interpolants.

Now you think convexity preserving interpolation is simple. Wrong, it isn’t. There is little known about it and nothing about edge cutting – except for the cutting schemes that we are developing right now. The 4-6-8 scheme is one such scheme, the honeycomb or the sqrt(3) cutting scheme are two more such methods. Since they are new, their characteristics can still be discovered and optimizing their free parameters is still open for research. And this is what this thesis is about.

Scope of the thesis
In this thesis, the student should implement at least one of the edge cutting schemes, study optimization techniques for related schemes like the umbrella scheme, transfer and adapt them to the implemented scheme,
test the results visualizing their shape qualities by reflection lines and curvatures. We anticipate that dualizing the schemes is a part of the thesis if it turns out to be useful. This should not be a big problem and is rather the opposite of a problem since dualization is no more than translating something into dual language, which can help to understand things better.

A particular question to be addressed is whether it is possible to obtain spheres from regular or arbitrary polyhedral circumscribed to spheres. This should be answered experimentally and optionally by theoretical investigations.

In the written thesis, the student should describe the field of corner cutting for curves, describe what we know about edge cutting for surfaces and how cutting schemes fit into the world of subdivision algorithms. Further, optimization techniques should be discussed and an assessment given for the ones chosen for this thesis.

Requirements
Programming experience in C++, Python or other OO-programming languages is mandatory. Experience with graphics programming and geometric modeling data structures like the half-edge structure for meshes helps but is not required. We do not expect the student to know subdivision or even corner cutting schemes. However, the student should have passed a course in numerics and should not shy away from mathematical and geometric reasoning.

Some links to subdivision
- https://www.youtube.com/watch?v=ckOTl2GcS-E
- https://www.youtube.com/watch?v=clcNrvq1fn4

Some related literature (ordered in decreasing relevance)

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