Skeletonizing the distance transform on the triangular grid

Skeletons are good representations of digital shapes and can be used for storage and shape editing. Skeletonization provides an effective and compact representation of an image object by reducing its dimensionality to a "medial axis" or "skeleton" while preserving the topologic and geometric properties of the object. The skeleton is useful for object description, retrieval, manipulation, matching, registration, tracking, recognition, compression, and it also facilitates efficient assessment of local object properties, e.g., scale, orientation, topology etc.

Weighted (or chamfer) distances on the triangular grid were introduced recently based on the three well-known neighborhoods (depicted below). The weighted distance between any two points along a path is given by a weight sequence. The weights are defined as a cost of movement between two neighbors, (e.g., in the grids shows below there are three types of neighbors 1, 2 and 3 of \( o \). The weight of movement from the origin to a neighbor of type 1, 2 or 3 is \( \alpha, \beta \) or \( \gamma \), respectively). The triangular grid, with three weights, overperforms the quality of the Euclidean distance approximation on the square grid by both two and three weights (i.e., by the traditional \( (3 \times 3) \)- and the \( (5 \times 5) \)-neighborhoods, respectively) in terms of maximal and average relative errors.

Studies of distance transformation and skeletonization were motivated by the need of converting a digital image into a linear form (a skeleton of a 2D object consists of 1D linear elements as a curve) in a natural manner.

Intuitively, the original binary image can be reconstructed by a union of the circular neighborhoods centered at each skeleton point with a radius equal to the associated label. Many algorithms have been developed to extract the skeleton in square grids. In this work we compute skeletons by a triangular weighted distance.

### Example of Skeletonization on some animals

<table>
<thead>
<tr>
<th>Beetle</th>
<th>bone</th>
<th>camel</th>
<th>steer</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Original" /></td>
<td><img src="image2.png" alt="Bone" /></td>
<td><img src="image3.png" alt="Camel" /></td>
<td><img src="image4.png" alt="Steer" /></td>
</tr>
<tr>
<td><img src="image5.png" alt="Skeleton" /></td>
<td><img src="image6.png" alt="Skeleton" /></td>
<td><img src="image7.png" alt="Skeleton" /></td>
<td><img src="image8.png" alt="Skeleton" /></td>
</tr>
</tbody>
</table>

### Types of neighborhood relations on the:

- **a) Square grid ((5 × 5)-neighborhoods)**
- **b) Triangular grid**

![Diagram](image9.png)

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