Karlsruhe Institute of Technology

Applied Geometry (CAGD)
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Curves and Surfaces in CAD Exercises — WS 2017

Homework Sheet 2

Exercise 2.1 (5 Points)

The **Bernstein operator** $\mathcal{B}$ assigns to a function f on $[0,1]$ the polynomial

$$\mathcal{B}[f](t) := \sum_{i=0}^{n} f(i/n) B_i^n(t).$$

Prove that if f is a polynomial of degree $m \leq n$, then $\mathcal{B}[f]$ is a polynomial of degree m.

Exercise 2.2 (4 Points)

Prove that the ith forward difference satisfies

$$\triangle^i \mathbf{b}_0 = \sum_{k=0}^{i} \binom{i}{k} (-1)^{i-k} \mathbf{b}_k.$$

Exercise 2.3 (6 Points)

Let $\mathbf{b}_0 = [2\ \ 0]^{\mathrm{t}}, \mathbf{b}_1 = [1\ \ 2]^{\mathrm{t}}, \mathbf{b}_2 = [4\ \ 3]^{\mathrm{t}}$ be the Bézier points of a curve $\mathbf{b}(u)$ with respect to the interval $[0,1]$.

1. Determine the Bézier representation of $\mathbf{b}(u)$ with respect to the interval $[1,3]$.
2. Determine the monomial representation of $\mathbf{b}(u)$.
3. Determine the Bézier representation of $\mathbf{b}(u)$ of degree 3 with respect to the interval $[0,1]$.

Exercise 2.4 (5 Points)

Let $\mathbf{b}(t) = \sum_{i=0}^{3} \mathbf{b}_i B_i^3(t)$ be a planar cubic Bézier curve with respect to $[0,1]$, where

$$\mathbf{b}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ \mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ \mathbf{b}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

For which $\mathbf{b}_3$ does $\mathbf{b}$ have a cusp, that is, $\dot{\mathbf{b}}(\hat{t}) = \mathbf{0}$ for some $\hat{t} \in [0,1]$?

Due Date: Friday, November 3, 2017, before the tutorial.